Atmospheric parameter estimation from AO wavefront sensing data: Application of the FADE method with NACO

Elisabeth Brunner[a], Julien H. Girard[b]

[a]Delft University of Technology, Mekelweg 2, Delft, The Netherlands;
[b]European Southern Observatory, Alonso de Córdova 3107, Santiago, Chile;

ABSTRACT

The performance of a ground based telescope can depend greatly on the coherence time, a site and time varying parameter determining the required reaction bandwidth of an Adaptive Optics system. Recently, Fast Defocus (FADE), a method which measures the coherence time from defocus fluctuations in a small telescope introduced by atmospheric turbulence has been presented. In this work, FADE was implemented for the Nasmyth Adaptive Optics System (NAOS) of the VLT's UT4 to demonstrate suitability for large scale telescopes. Estimates of the coherence time for an examplary set of AO loop data were compared to results obtained with the DIMM and MASS instruments, showing good agreement which justify a future in depth analysis for large AO loop data samples provided by NAOS as well as the VLT's Spectro-Polarimetric High-contrast Exoplanet Research (SPHERE) instrument.

Keywords: Atmospheric turbulence, Coherence Time, Adaptive Optics, Wavefront Sensing, Site Testing.

1. INTRODUCTION

Next to the seeing, the performance of an Adaptive Optics (AO) assisted ground based telescope - aiming for the diffraction limit - can be characterised by the coherence time, a site and time varying parameter which determines the required reaction bandwidth of an AO system.[1] The coherence time depends on the vertical profile of the turbulence's wind velocity which is not available in real-time. Hence a practical method to estimate the coherence time is necessary.

A working AO system provides real-time data on the wavefront evolution and the temporal structure function of Zernike modes introduced by atmospheric turbulence can serve to derive the coherence time. Fusco et al.[2] presented a method, estimating the coherence time from half-time correlation of Zernike aberration, which has been implemented at the VLT AO instrument NAOS at Paranal, but is only valid for a single layer of atmospheric turbulence. Recently, Tokovinin and Kellerer[3,4] proposed the method Fast Defocus (FADE) which measures the coherence time from defocus fluctuations and is based on a multi layer turbulence profile. Sequences of fast defocus measurements are processed to obtain the temporal structure function, which is fitted with a model to derive estimates of the wind velocity and the Fried parameter of the turbulence. These parameters allow the computation of the seeing as well as the coherence time. FADE was introduced as a stand-alone monitor using a 36-cm telescope[3] and tested for AO loop data of a 4m telescope with SOAR Adaptive Module (SAM).[5,6]

This work is the first step of an effort to implement FADE for NAOS in order to demonstrate the suitability of the method for large scale telescopes. The coherence time was computed directly from AO loop data using the deformable mirror (DM) voltages provided by Adaptive Optics NIR Instrument NAOS-CONICA (NACO).[7,8] Details of this implementation which are specific for NACO are described in Section 3 following a short overview on FADE in Section 2. Estimates of the coherence time were obtained with FADE for a small number of data sets and shall be seen as an indicator for future testing with a larger number of samples. In Section 3.3, FADE estimates for AO loop data recorded in three different nights are compared with the simultaneous results provided by the Differential Image Motion Control (DIMM)[9] and the Multi Aperture Scintillation Sensor (MASS)[10] which are both installed at Paranal.

Further author information: (Send correspondence to J.G.)
E.B.: E-mail: A.E.Brunner@tudelft.nl
J.G.: E-mail: jgirard@eso.org

2. COHERENCE TIME ESTIMATION WITH FADE

In this section, we give a short explanation of the FADE technique[3,4] which is based on recording and processing defocus fluctuations produced by the atmospheric turbulence in order to retrieve the atmospheric parameters Fried parameter r_0 and average wind speed $\bar{V}$, from which the coherence time τ_0 defined in Section 2.1 can be computed. Wavefront distortions are commonly decomposed into Zernike modes. FADE is based on fitting a model $D_4^m(t)$ for time t, which will be introduced in Section 2.3, of the temporal structure function (SF) of the defocus coefficient in the modal representation of the wavefront to a sequence of structure function values $D_4(t)$ computed from AO loop data (see Section 2.2).

2.1 Definition of atmospheric parameters

First, we introduce the relevant atmospheric parameters and the AO time constant τ_0. The standard definition of the coherence time is

$$\tau_0 = 0.314\frac{r_0}{\bar{V}} = \left(118\lambda_0^{-2}\int_0^\infty \mathrm{C}_n^2(h)(V(h))^{\frac{5}{3}}\right)^{-\frac{3}{5}}, \tag{1}$$

with $\mathrm{C}_n^2(h)$ denoting the refractive index structure parameter at altitude h and λ_0 the observed wavelength. In a multi-layer turbulence distribution a mean propagation velocity with modulus $\bar{V}$ is assumed and given by

$$\bar{V} = \left(\frac{\int_0^\infty (V(h))^{\frac{5}{3}}\mathrm{C}_n^2(h)dh}{\int_0^\infty \mathrm{C}_n^2(h)dh}\right)^{\frac{3}{5}}. \tag{2}$$

The Fried parameter r_0 is obtained in terms of the integrated turbulence with

$$r_0 = \left[0.423\left(\frac{2\pi}{\lambda_o}^2(\cos(\alpha))^{-1}\int_0^\infty \mathrm{C}_n^2(h)dh\right)\right]^{-\frac{3}{5}}, \tag{3}$$

where α is the zenith angle of the observation, i.e. the angular distance of the source from the zenith.[1]

2.2 Defocus measurements from AO loop data

For a sequence $a_4(t)$ of measured defocus Zernike coefficients of the wavefront's modal representation, we obtain values of its temporal structure function $D_4(t)$ at time instant t by computing

$$\begin{aligned} D_4(t) &= \left\langle \left(a_4(t'+t) - a_4(t')\right)^2 \right\rangle \\ &= 2(R_4(0) - R_4(t)), \end{aligned} \tag{4}$$

where $\langle \cdot \rangle$ denotes the exptected value. The temporal autocorrelation function $R_4(t)$ of $a_4(t)$ is given by

$$\begin{aligned} R_4(t) &= \langle\, a_4(t')a_4(t'+t) \,\rangle \\ &= \mathcal{F}^{-1}\left(|\mathcal{F}(a_4(t))|^2\right) \end{aligned} \tag{5}$$

with the Fourier transform and its inverse by $\mathcal{F}$, $\mathcal{F}^{-1}$. Note that this computation includes the calculation of

$$W_4(f) = |\mathcal{F}(a_4(t))|^2 \tag{6}$$

which is the temporal power spectrum of defocus coefficient sequence $a_4(t)$ defined for frequencies f.[1]

2.3 Structure function model and estimation of τ_0

Kellerer and Tokovinin[3,4] proposed the following model of the defocus structure function for N turbulent layers in terms of the Fried parameters $r_{0,i}$ and the wind speeds V_i in the respective layers $i = 1, ..., N$:

$$D_4^m(t) = 1.94 D^{\frac{5}{3}} \sum_{i=1}^{N} r_{0,i}^{-\frac{5}{3}} K_4(2tV_i/D, \epsilon) + n\,, \tag{7}$$

where D is the diameter of the telescope pupil, n the noise paramter and ϵ the relative central obstruction. An approximation of function $K_4(\beta, \epsilon)$ was introduced as

$$K_4(\beta, \epsilon) \approx \frac{C_1\beta^2 + C_2\beta^6}{1 + C_3\beta^\alpha + \beta^6}\,, \tag{8}$$

which is valid for central obscurations $\epsilon < 0.6$. Coefficients C_i and index α in Equ. (8) are cubic polynomials in ϵ defined in Tokovinin (2008).[3]

In order to retrieve the atmospheric parameters $r_{0,i}$ and V_i, the model $D_4^m(t)$ is fitted to a sequence $D_4(\boldsymbol{t})$ of measured SF values computed with Equ. (4) for time instants $\boldsymbol{t} \in \mathbb{R}^M$. The fit is applied to the initial part of the structure function, up to the time incremement t_{fit}. Uambiguous fitting of the parameters is guaranteed as long as $t_{\text{fit}} > \frac{1}{f_{\text{fr}}}(2N+1)$ for a SF model $D_4^m(t)$ of N layers and an acquisition frequency f_{fr} for the wavefront sensor. The time scale of the defocus SF is defined by the half-width $t_{0.5}$ of auto-correlation function $R_4(t)$ which is 0.3 times the aperture crossing time, hence $t_{0.5} = 0.3D/V$ for wind speed V, setting an upper limit to the time lag t_{fit} for the model-fitting.

After obtaining the estimates for $r_{0,i}$ and V_i of each layer $i = 1, ..., N$, the average atmospheric parameters r_0 and $\bar{V}$ can be computed with relations

$$r_0^{-\frac{5}{3}} = \sum_{i=1}^{N} r_{0,i}^{-\frac{5}{3}} \quad \text{and} \quad \left(\frac{\bar{V}}{r_0}\right)^{\frac{5}{3}} = \sum_{i=1}^{N} \left(\frac{V_i}{r_{0,i}}\right)^{\frac{5}{3}}, \tag{9}$$

and the FADE estimate of coherence time τ_0 is obtained with the left equality of Equ. (1).[3]

The FADE method is based on a multi layer SF model taking into account several layers with different wind speeds. As the defocus is rotationally symmetric and hence indpendent of the wind direction, a true C_n^2-weighted estimator is obtained.[4]

3. FADE FOR NACO LOOP DATA

The goal of this work was to implement FADE for AO loop data of the Nasmyth Adaptive Optics System (NAOS) installed at the UT4 of the VLT.

3.1 Implementation details

For this paper, we used NACO data was which was recorded in several nights between May 2012 and August 2013 with the new GRAB button which enables us to save RTC telemetry data (up to 4096 consecutive slopes slopes or $\sim$ 9 seconds of data at 444 Hz) simultaneously with high frame - cube mode - CONICA images.[11] Defocus fluctuations were tracked from signals of the DM accounting for the frequency response of the closed AO loop. The UT4 has a pupil diameter D of 8m with a central obscuration ratio of $\epsilon = 0.14$. For the used data sets, the equivalent observed wavelength is $\lambda_0 \sim 750$nm (the WFS is sensitive from 600 to 900 nm) and the acquisition frequency of the wavefront sensor varied between 444Hz or 480Hz. Taking the wind speed of 20m/s, one obtains $t_{0.5} = 120$ms, which is well superior to the temporal sampling $dt = 1/f_{\text{fr}} < 2.3$ms and equivalent to $\sim$ 50 loop cycles of NACO. Note, that although the acquisition frequency is high, we are limited by the AO frequency of only $\sim f_{\text{fr}}/10 = 44$ Hz which causes a strong correlation of the points in the SF. The option to speed up FADE by using pseudo-open-loop data instead of DM voltages is planned for a future paper.

Samples of the defocus structure function were obtained by transforming DM voltages into Zernike coefficients in radians at λ_0 and postprocessing those to zero mean defocus time sequences. The temporal power spectrum $W_4(f)$ of the time sequences was computed with Equ. (6). An example, of the power spectrum of a data set acquisioned at 444Hz is plotted in log-log scale on the left in Figure 1. A plot of the low frequency range of the power spectrum in normal scale on the right shows clearly vibration lines at 45 Hz and 48 Hz. As instrumental effects should be excluded from the measurement of τ_0, we suppress all frequencies above 44 Hz. Since NAOS does not follow aberrations of temporal frequencies higher than $\sim$ 44 Hz, as mentioned earlier, this low-pass filtering has little effect on the results. With the truncated power spectrum, we carry on to compute the autocorrelation function $R_4(t)$ and subsequently the required SF values $D_4(\boldsymbol{t})$ with Equ. (4) for time instants $\boldsymbol{t} \in \mathbb{R}^M$, where $t_j = j(1/f_{\text{fr}})$, $j = 1, ..., M$.

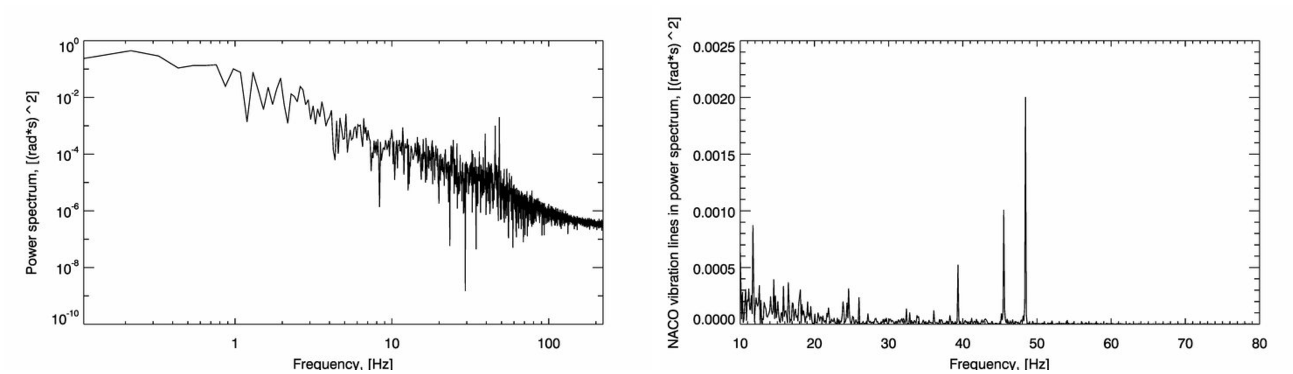

Figure 1. Left: Temporal power spectrum in log-log scale computed from a defocus coefficient sequence of NACO DM voltages recorded with a frequency of 444 Hz. Right: Low frequency range of the same power spectrum displaying the NACO vibration lines at 45 Hz and 48 Hz.

To perform the FADE method as presented in Section 2.3, the model $D_4^m(t)$ of the defocus SF introduced in Equ. (7) is evaluated at the same time instances $\boldsymbol{t}$ and fitted to the measured SF samples by minimizing the cost function

$$\min_{r_{o,i}, V_i, n} \| D_4(\boldsymbol{t}) - D_4^m(\boldsymbol{t}) \|_2^2 \,, \tag{10}$$

where $\|\cdot\|_2^2$ denotes the 2-norm. The least-squares optimisation problem was solved with the Levenberg Marquardt procedure in IDL, where upper and lower boundaries were set to {0.1m,10m} for the Fried parameters $r_{0,i}$ and to {0.1m/s,100m/s} for the windspeeds V_i in each layer i. For the noise parameter n which adds another degree of freedom to the problem the limits were set to $\{0,\, D_4^m(M(1/f_{\text{fr}}))\}$.

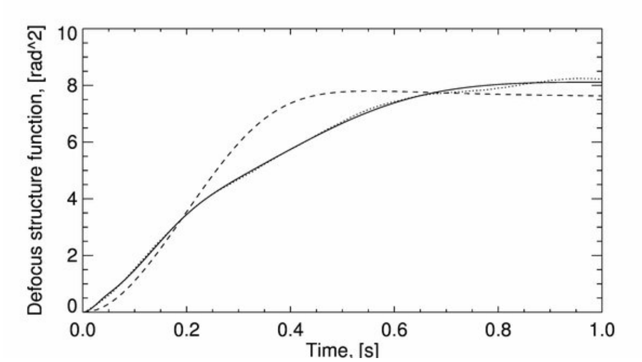

Figure 2. Defocus SF values computed from example data sets over a time increment of ≈1s (dotted line) next to SF model of N = 3 layers evaluated at same time instances for guessed initial parameters (dashed line) and for fitted parameters (solid line).

3.2 Data observations

Figure 2 shows the SF of the defocus measured with NAOS in May 2012 (dotted line), next to the theoretical model $D_4^m(t)$ of the defocus SF. The solid line shows the values of $D_4^m(t)$, over the used time increment of 1s, after fitting with the measured data, the dashed line plots the same model for guessed initial values of parameters $r_{i,0}$, V_i and n. We consider a strong and slow ground layer which extends the defocus SF to $\sim$ 1s instead of $0.3D/V = 120$ms. For given sampling time $dt \sim 2.3$ms this time lag corresponds to a fit with around 145 instead of 54 data points.

In Figure 3, we consider the influence of the maximum time lag t_{fit} used in the model fitting and the effect of a higher number N of layers in the SF model for 7 data samples recorded in four nights between May 2012 and August 2013. It shows in the plot on the left, that increasing the lag from $t_{\text{fit}} = 120$ms (fit with 54 data points) to $t_{\text{fit}} = 1$s (fit with 145 data points) results in larger estimates $\tau_{0,\text{fade}}$. The differences observed when the FADE results for a 3 layer model are compared to those obtained with a 6 and 13 layer model however are very small for fitting with 54 data points as can be seen on the right. For 145 data points, the effect of increasing the number of layers in the model was non significant (not depicted here). For the results in the following section, we apply FADE for the extended time lag of $t_{\text{fit}} = 1$s and a 3 layer SF model.

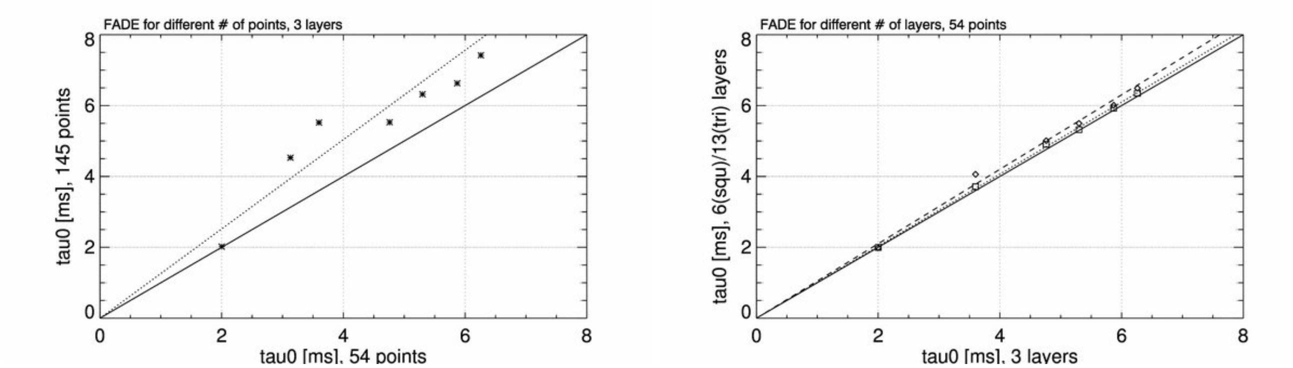

Figure 3. Left: Relation between $\tau_{0,\text{fade}}$ for 54 data points (x-axis) and 145 data points (y-axis); the full line corresponds to equality, the dotted line has the average slope 1.26. Right: Relation between $\tau_{0,\text{fade}}$ for a 3 layer model (x-axis) and a 6 layer model (y-axis, squares) as well as a 13 layer model (y-axis, diamonds); the full line corresponds to equality, the dotted line has the average slope 1.02 for 6 layers, the dashed line 1.05 for 13 layers.

3.3 Comparison with DIMM and MASS

In this section, the results of FADE computed from NACO loop data are compared to corresponding values of the DIMM and MASS database at Paranal. The DIMM instrument provides at most a crude approximation $\tau_{0,\text{dimm}}$ of the coherence time which is empirically derived from a combination of DIMM seeing[9] and wind velocity at 200mb (height of the jet stream) and at the ground.[12] The MASS instrument[13] was designed for the vertical profiling of atmospheric turbulence derived deduced from scintillation and further provides a meausure of the coherence time of the higher layers. The MASS values recorded at the Paranal database[14] are based on the differential-exposure index, DESI.[5] It was shown that a correction factor $C \sim 1.7$ (Marc Sarazin, private com.) has to be applied, to compensate for a bias in the DESI estimates $\tau_{0,\text{desi}}$. Therefore, for the following comparison, we consider $\tau_{0,\text{mass}} = 1.7\,\tau_{0,\text{desi}}$.[15,16]

Figure 4 shows the coherence time estimates $\tau_{0,\text{fade}}$ for NACO loop data recorded in three different nights between February 2012 and August 2013 obtained with FADE, where a 3 layer SF model was fitted to the data for a time lag of 1s which corresponds to 145 SF points. To obtain the simultaneous values for the DIMM and MASS instruments, a crude average value of the results at the closest time instants in the database was used. On the left plot, one can see a comparison with simultaneous values provided by DIMM showing good correlation but an average difference of 28%. For the comparison with MASS estimates (multiplied by factor 1.7) in the right plot, we obtain similar correlation with the FADE estimates, but the results deviate in average only by 9%. The different nights for which data was recorded can be clearly distinguished as three groups of points, with bad

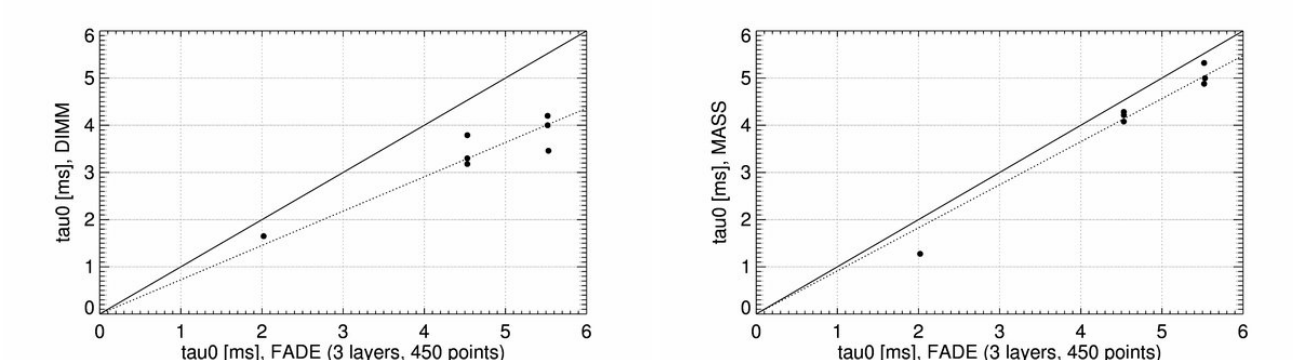

Figure 4. Coherence time derived from the recorded data with FADE by fitting a model with N = 3 layers for fitting time t_{fit} = 1s (x-axis) compared to simultaneous DIMM results (left) and to measurements provided by MASS (right). The full lines corresponds to equality, the dotted line has the average slope 0.72 for DIMM and 0.91 for MASS.

conditions of approximately 2ms of coherence time for the first night. For the other two nights loop data was recorded three times within a few minutes leading to almost identical values for FADE.

4. CONCLUSION AND FUTURE WORK

Fast Defocus (FADE), a method for coherence time estimation from defocus fluctuations in atmospheric turbulence recorded by a ground-based telescope, was implemented for the Nasmyth Adaptive Optics System (NAOS) at the VLT. Samples of the defocus structure function (SF) were computed from postprocessed DM voltages and fitted to a theoretical expression of the SF given by a multi layer based model. With FADE an alternative to Fusco's method for coherence time estimation from AO loop date is obtained, where FADE in opposite to the latter is valid for a turbulence of several layers. For the current implementation for DM voltages, FADE for NAOS is restricted by the AO response frequency of the system which is far below the sampling speed of the wavefront sensor. Hence we do not benefit from the larger pupil diameter and the high acuisition frequency. This could be resolved by the usage of pseudo open loop instead of closed loop data promising a major speed up of the method.

A small number of data sets recorded in three different nights was used to compute FADE estimates of the coherence time, which does not allow a thorough analysis but gives indication for future more extensive testing. The FADE results were computed for different model and fitting configurations and compared to the simultaneous values saved in the DIMM-MASS database of Paranal. Next to good correlation with the empirical measure provided by the DIMM instrument, the main positive observation was the good agreement with the MASS results. The MASS method is also based on modelling the vertical profile of the atmospheric turbulence and hence a more reliable method for comparison.

Encouraged by these results further tests with large data samples for fine tuning of the method are planned. A detailed analysis of the correlation between FADE estimates of the coherence time against MASS results and external parameters will allow a full understanding of the potential of the FADE method for large scale telescopes like the VLT. Next to AO loop data provided by NAOS, the application of FADE for SPHERE wavefront sensing data which can be recorded with 1200 Hz at a frequency around three times as big can be a second stage of the analysis.

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